

The Schrödinger equation in binocular brightness combination

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Abstract. Schrödinger once proposed a simple rule to describe how binocular brightness depends upon the inputs from left and right eyes in situations where the two eyes receive different light intensities. It is shown that Schrödinger's suggestion can account for the main features of Levelt's recent data if it is assumed that each monocular response varies linearly with the logarithm of the light intensity.

When geometrically similar stimuli of different intensities are presented one to each eye, they may fuse in vision. When this happens the brightness of the binocular percept may be intermediate between the two monocular brightnesses, or it may be greater than either of them. The relation between the monocular and binocular brightnesses exhibits two problematical features which any quantitative treatment of the phenomenon must account for.

First, there is Levelt's (1965) finding that, if the physical intensity of one stimulus is increased by a moderate amount, the intensity of the stimulus delivered to the other eye must be decreased by an equal (or a proportionate) amount for the binocular brightness to remain constant: in an 'equibrightness curve' (figure 1) left eye luminance is almost linear with right eye luminance along most of the curve. This

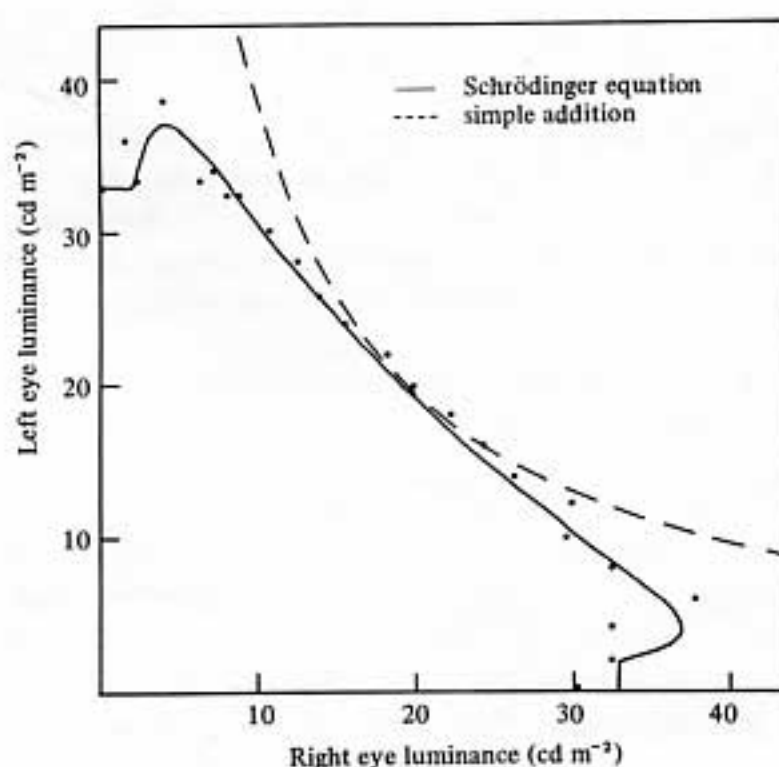


Figure 1. An equibrightness curve, showing left eye luminance and right eye luminance for binocular stimuli of equal brightness. The points are a representative set of data from Levelt (1965, figure 8a).

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linearity could be understood by supposing that the brain determines binocular brightness by adding together two neural effects, one originating from each eye, and each varying linearly with the monocular luminance that caused it. But it is likely, for reasons given below, that the monocular effects with which the brain must deal are strongly nonlinear with luminance. If they are, then a nonlinear equibrightness curve must be expected⁽¹⁾, and the observed linear relation contradicts the assumption that the monocular effects are simply added.

A second problem is posed by the positive slopes of the equibrightness curves at extreme intensity ratios where the linearity breaks down. There, a constant binocular brightness can only be maintained by increasing or decreasing both monocular luminances together. If this is to be explained by addition of monocular effects, one of the effects must be decreasing with luminance; yet monocular brightness always increases with increasing luminance if nothing else changes.

To cope with this second problem (long familiar as Fechner's paradox) Schrödinger in 1926 (during what he later described as a brief escape from the difficulties of fundamental physics) suggested that binocular combination does not take place by simple addition, and proposed instead an attractive nonadditive description. His suggestion was that when the monocular effects are combined each eye is assigned a weight equal to the ratio of its effect to the sum of the two:

$$B = \psi_L \frac{\psi_L}{\psi_L + \psi_R} + \psi_R \frac{\psi_R}{\psi_L + \psi_R}, \quad (1)$$

where ψ_L and ψ_R are the monocular effects and B is the binocular result⁽²⁾. When ψ_L is much less than ψ_R the term on the left is unimportant by comparison with the term on the right, so that an increase in ψ_L will bring about a decrease in B even though B is directly proportional to ψ_L in the absence of ψ_R . In this way Fechner's paradoxical dimming is accounted for.

If curves of constant B in equation (1) are set out with ψ_L and ψ_R as axes, each curve is a circle passing through the origin. If ψ_L and ψ_R varied linearly with their respective luminances, equibrightness curves like the one in figure 1 would also lie on circles (or ellipses). Though qualitatively successful with Fechner's paradox, Schrödinger's scheme would then fail to describe the linear part of the equibrightness curve. But the linear part is after all quite easily accounted for on a simple addition basis if ψ_L and ψ_R are linear with luminance: the problem is to reconcile the linear equibrightness curve with a presumed nonlinear (perhaps logarithmic) relation between the monocular effects and luminance. Equation (1) permits this reconciliation as the following application of it will show.

First, the way that the monocular effects vary with luminance must be specified. For this the following equation will be used:

$$\begin{aligned} \psi &= \psi_0 + \log(\phi/\phi_0) & \text{for } \phi \geq \phi_0 \\ \psi &= \psi_0 & \text{for } \phi \leq \phi_0, \end{aligned} \quad (2)$$

where ψ is the size of a monocular effect, ϕ is the difference in luminance across the contour of the corresponding monocular stimulus, ϕ_0 is the threshold luminance difference, and ψ_0 is the contribution of spontaneous activity. The unit of measurement for ψ is the change in ψ brought about by a tenfold change in ϕ .

⁽¹⁾ Leaving aside the special cases in which the function relating size of effect to luminance is nonlinear with odd symmetry about the 'standard' luminance, which in Levelt's experiments produced the required brightness when viewed with both eyes.

⁽²⁾ I have encountered the same suggestion in an unpublished manuscript (ca 1940) by K. J. W. Craik.

The identification of ϕ with a luminance difference rather than with absolute luminance is required by some results of Fry and Bartley (1933, experiments 3 and 5) and Levelt (1965, chapter 4), as well as by the pervasiveness of centre-surround antagonism in the visual system. The form of equation (2) has been decided on the basis of the following evidence.

First, a Fechnerian argument demands it. If it is assumed that a just noticeable difference in intensity corresponds to a fixed increment in the neural effect, the observed relation between flash intensity and the just noticeable difference in intensity requires that the neural effects approximately obey equation (2): the just noticeable difference in ϕ decreases until the brighter flash reaches approximately ϕ_0 and thereafter increases in proportion to ϕ (Cornsweet and Pinsker, 1965; Whittle and Swanston, to be published). Contrast discrimination for steadily exposed gratings of a fixed average luminance behaves in the same way as flash discrimination (Campbell and Kulikowski, 1966); ϕ is here proportional to grating contrast.

Second, Campbell and Maffei (1970) and Campbell and Kulikowski (1972) found that the visually evoked cortical potential varies in accordance with equation (2).

Some difficulties remain in applying equation (2). For instance, it is not clear how far one can generalize from one type of stimulus to another, so that ideally one would have equibrightness curves for flashes or gratings. Again, in the experiments cited in support of equation (2) the state of light adaptation was probably not much dependent on ϕ . But in Levelt's experiments the state of adaptation was not controlled.

The equation of an equibrightness curve is obtained by substituting equation (2) for each eye into equation (1). The shape of the curve depends mainly on the brightness parameter B . The threshold luminance parameter ϕ_0 fixes the dimensions of the curve (but not its shape) when the axes plot luminance. The parameter ψ_0 affects mainly the ends of the curve, the parts where one of the luminances involved is less than ϕ_0 . The greater the value of ψ_0 , the greater is the luminance required in monocular observation to match a given binocular luminance.

The continuous curve of figure 1 was obtained with $B = 1.36$, $\psi_0 = 0.34$, and $\phi_0 = 2.0 \text{ cd m}^{-2}$ for each eye, but the shape of the curve is not very sensitive to changes in these values. The dotted line is the curve predicted if monocular effects obeying equation (2) simply add together. The fit given by the solid line is satisfactory and is comparable to the excellent fit achieved by Engel (1969) using a very much more complicated model.

It appears then that binocular brightness combination, though it does not take place by simple addition of effects, does approximately follow the hardly less simple rule suggested by Schrödinger. The natural question then is: what physiological mechanisms might implement the Schrödinger equation? Microelectrode investigations may already have laid the foundation for an answer: it is now well established (Henry *et al.*, 1969) that in certain cells of the primary visual area of the cat a discharge evoked by stimulation of the dominant eye will be partially suppressed, rather than increased, if the stimulus is delivered also to the other eye. This could be the basis of Fechner's paradox, although there is not yet enough quantitative evidence to make the analogy a close one.

Alongside the physiological explanation it is possible to put forward a 'weaselly teleological reason' (Walls, 1963, p.332) for the nonadditive properties of binocular interaction. If binocular combination were additive, vision using only one eye would differ very noticeably from vision with both eyes, as Asher (1961) has pointed out. But the Schrödinger equation makes monocular and binocular responses similar. This is no doubt a valuable advantage, especially since in most animals (including man) the binocular visual field is flanked by monocular fields continuous with it on either side.

References

- Asher, H., 1961, *The Seeing Eye* (Duckworth, London).
 Campbell, F. W., Kulikowski, J. J., 1966, "Orientational selectivity of the human visual system", *Journal of Physiology*, 187, 437-445.

- Campbell, F. W., Kulikowski, J. J., 1972, "The visual evoked potential as a function of contrast of a grating pattern", *Journal of Physiology*, 222, 345-356.
- Campbell, F. W., Maffei, L., 1970, "Electrophysiological evidence for the existence of orientation and size detectors in the human visual system", *Journal of Physiology*, 207, 635-652.
- Cornsweet, T. N., Pinsker, H., 1965, "Luminance discrimination of brief flashes under various conditions of adaptation", *Journal of Physiology*, 176, 294-310.
- Engel, G. R., 1969, "The autocorrelation function and binocular brightness mixing", *Vision Research*, 9, 1111-1130.
- Fry, G. A., Bartley, S. H., 1933, "The brilliance of an object seen binocularly", *American Journal of Ophthalmology*, 16, 687-693.
- Henry, G. H., Bishop, P. O., Coombs, J. S., 1969, "Inhibitory and sub-liminal excitatory receptive fields of simple units in cat striate cortex", *Vision Research*, 9, 1289-1296.
- Levelt, W. J. M., 1965, *On Binocular Rivalry* (Institute for Perception RVO-TNO, Soesterberg).
- Schrödinger, E., 1926, "Die Gesichtsempfindungen", in *Mueller-Pouillet's Lehrbuch der Physik* (11th ed.), Book 2, Part 1 (Vieweg, Braunschweig), pp.456-560, especially 558-560.
- Walls, G. L., 1963, *The Vertebrate Eye* (2nd ed.) (Hafner, New York).
- Whittle, P., Swanston, M. (to be published), "The discrimination of two simultaneous non-adjacent light flashes and its relation to adaptation and to subjective brightness".