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NOTE

Remarks on the Constant Luminance Chromaticity Diagram

Donald I. A. MacLeod
Robert M. Boynton

Department of Psychology
University of California, San Diego
La Jolla, California 92093

We hope that readers of Fry's article[1] will view it in the light of the following remarks, which are intended to correct some misconceptions in his account of the constant luminance chromaticity diagram we have advocated.[2]

Accepting, as we always have[2] and still do, Judd's choice of the tristimulus value $\bar{y}'$ to represent luminance, it follows that at zero luminance, $\bar{y}' = 0$ and thus the alychne, or locus of zero-luminance stimuli in the (x', y') chromaticity diagram, will be the x' axis; only along that axis is $y' = \bar{y}'/(\bar{x}' + \bar{y}' + \bar{z}') = 0$. We have made no proposal that would alter the location of the alychne in Judd's diagram. That location is fixed by the identification of $\bar{y}'$ with luminance.

We do, as Fry indicates, make the popular assumption that the short wavelength sensitive cones ("blue cones") make no contribution to luminance. That assumption will be correct if and only if luminance is determined solely by the *other* cone excitations, R and G, regardless of the blue cone excitation B. In Fry's eqs. (1)–(3) (which are from our article,[2] but are cited here using the numbering of Ref. 1), this requirement is satisfied, since luminance ($\bar{y}'$) is given simply by the unweighted sum of R and G as defined by eq. (1) and (2), respectively:

$$\bar{y}' = R + G$$

It is critical to recognize that Fry's eq. (3), defining blue cone excitation, has no bearing whatever on our assumption that blue cones do not contribute to luminance (or in other terms, that the blue fundamental has zero luminosity). In deciding whether luminance is or is not determined by the pair of quantities R and G, the behavior of some third quantity, be it blue cone excitation or the price of eggs, is simply not relevant. Fry is therefore mistaken in his contention that "If we were to allow the x' axis to be the alychne and allow the blue primary to have a luminosity coefficient of zero, the coefficient of $\bar{z}'$ in eq. (3) would have to be zero." Only eqs. (1) and (2) are relevant to our assumption, and these, as we have just seen, are perfectly consistent with it.[3]

Our proposed chromaticity coordinates $(R/(R + G), B/(R + G))$ represent colors by the cone excitations they evoke at constant luminance.[2] The effect of our choice of the scaling factor 0.01608 for blue cone excitation, B, in eq. (3) is merely to fix the vertical scale of the diagram without changing the representation in any other way: both coordinates remain proportional to cone excitation at constant luminance. The property that scaling B merely scales the chromaticity coordinates is unique to the constant luminance diagram. Geometrically, it arises because the plane of the chromaticity diagram is invariant, being in all cases parallel to the B axis in cone excitation space; algebraically, it arises because the conversion factor $1/(R + G)$ relating the coordinates to the cone excitations does not depend on B.

In his eqs. (4)–(6), and in the corresponding chromaticity diagram (his Fig. 1), on which the remainder of his discussion is based, Fry follows a path quite different from ours.[2] The denominator on both sides of those equations is $R + G + B$. Thus his Fig. 1 is derived by a projection onto the unit triangle $R + G + B = 1$. Since the blue cone excitation B is generally not zero, this is not a plane of constant luminance. Fry's discussion based on his eqs. (4)–(6) has no bearing, so far as we can determine, on the proposals of our paper.[2]

In the constant luminance chromaticity diagram[2] (Fry's Fig. 5), the denominator in Fry's eqs. (4)–(6) is replaced by $R + G$ and so the projection is onto the plane $R + G = 1$. Tritanopic confusion lines are then parallel (vertical), and the imaginary blue fundamental stimulus is at infinity; but real stimuli do have finite coordinates. The parallelism

 CCC 0361-2317/89/030157-02$04.00

of the tritanopic lines is not a "problem" to be "solved" by tinkering with coefficients as Fry suggests. It is a welcome simplification: it is what enables the horizontal axis to represent red or green cone excitation directly, independently of blue cone excitation.

1. G. A. Fry, Color vision model of MacLeod and Boynton, *Color Res. Appl.*, **14,** 152–156 (1989).
2. D. I. A. MacLeod and R. M. Boynton, Chromaticity diagram showing cone excitation by stimuli of equal luminance, *J. Opt. Soc. Am.* **69,** 1183–1187 (1979). A diagram similar to the one we proposed was introduced by R. Luther, Aus dem Gebiet der Farbreizmetrik, *Zeitschr. f. techn. Physik* **8,** 540–558 (1927). Since the CIE diagram did not then exist, it fell to us to draw attention to the comparative advantages of Luther's diagram 50 years later.
3. Like Fry, we focus here on the colorimetric consequences of the assumption rather than on its empirical status. But it is worth noting that considerable experimental support for it has been added to the work we cited in Ref. 2. See A. Eisner and D. I. A. MacLeod, Blue-sensitive cones do not contribute to luminance, *J. Opt. Soc. Am.* **70,** 121–123 (1980); also P. Cavanagh, D. I. A. MacLeod and S. M. Anstis, Equiluminance: spatial and temporal factors and the contribution of blue-sensitive cones, *J. Opt. Soc. Am.* **A4,** 1428–1438 (1987). There is recent evidence that blue cones do contribute (weakly and negatively) to luminance under adaptation to extremely intense long-wavelength backgrounds: A. Stockman and D. I. A. MacLeod, An inverted S-cone input to the luminance channel: evidence for two processes in S-cone flicker detection, *Invest. Ophthal. Vis. Sci (suppl.)* **28,** 92 (1987). But even this recent work upholds the previous evidence that the contribution is practically negligible under the sorts of conditions generally encountered.

Author's reply:

In the note "Remarks on the Constant Luminance Chromaticity Diagram," MacLeod and Boynton say they want to correct misconceptions in my article, "The Color Vision Model of MacLeod and Boynton," about the constant luminance chromaticity diagram they have advocated.

In their first paragraph they claim that they have made no proposal that would alter the location of the alychne on Judd's diagram. And, to be sure, they did not intend to do so and were not aware of doing it; however, by defining

$$B = 0.01608\,\bar{z}'$$

they have inadvertently changed the position of the alychne. This is explained in my article.

In addition, they have used the transformation equations (1), (2), and (3) to derive the cone response curves shown in Fig. 3. Because adding eqs. (1) and (2) yields

$$\bar{y}' = R + G$$

they have concluded that luminance equals the sum of R and G. But I have made no claim that luminance does not equal the sum of R and G. All that I claim is that they have used the wrong approach to arrive at this conclusion.

Equations (27), (28), and (29) are the proper equations for computing R, G, and B and the derivation of these equations is fully explained.

Equations (1), (2), and (3) give the right answers. It would be helpful if MacLeod and Boynton would explain how these equations are derived.

Glenn A. Fry
College of Optometry
The Ohio State University
Columbus, Ohio 43210